

What Is Claimed Is:

- 1 1. A method for using a computer system to solve a global
2 optimization problem specified by a function f and a set of equality constraints,
3 the method comprising:
4 receiving a representation of the function f and the set of equality
5 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) at the computer system, wherein f is a scalar
6 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
7 storing the representation in a memory within the computer system;
8 performing an interval global optimization process to compute guaranteed
9 bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the set of
10 equality constraints;
11 wherein performing the interval global optimization process involves,
12 applying term consistency to the set of equality constraints
13 over a subbox \mathbf{X} , and
14 excluding portions of the subbox \mathbf{X} that can be shown to
15 violate any of the equality constraints.
- 1 2. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 preconditioning the set of equality constraints through multiplication by an
4 approximate inverse matrix \mathbf{B} to produce a set of preconditioned equality
5 constraints;
6 applying term consistency to the set of preconditioned equality constraints
7 over the subbox \mathbf{X} ; and
8 excluding portions of the subbox \mathbf{X} that can be shown to violate any of the
9 preconditioned equality constraints.

1 3. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;
4 unconditionally removing from consideration any subbox for which
5 $\inf(f(\mathbf{x})) > f_bar$;
6 applying term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} ;
7 and
8 excluding portions of the subbox \mathbf{X} that violate the inequality.

1 4. The method of claim 1, wherein applying term consistency
2 involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term, $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$,
5 wherein the term $g(x_j)$ can be analytically inverted to produce an inverse function
6 $g^{-1}(y)$;
7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(X'_j) = h(\mathbf{X})$;
9 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
10 intersecting X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
11 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
12 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
13 the size of the subbox \mathbf{X} .

1 5. The method of claim 1, wherein performing the interval global
2 optimization process involves:

3 applying box consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$
4 ($i=1, \dots, r$) over the subbox \mathbf{X} ; and
5 excluding portions of the subbox \mathbf{X} that violate the set of equality
6 constraints.

1 6. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 evaluating a first termination condition;
4 wherein the first termination condition is TRUE if a function of the width
5 of the subbox \mathbf{X} is less than a pre-specified value, ε_X , and the absolute value of the
6 function, f , over the subbox \mathbf{X} is less than a pre-specified value, ε_F ; and
7 if the first termination condition is TRUE, terminating further splitting of
8 the subbox \mathbf{X} .

1 7. The method of claim 1, wherein performing the interval global
2 optimization process involves performing an interval Newton step on the John
3 conditions.

1 8. A computer-readable storage medium storing instructions that
2 when executed by a computer system cause the computer system to perform a
3 method for using a computer system to solve a global optimization problem
4 specified by a function f and a set of equality constraints, the method comprising:
5 receiving a representation of the function f and the set of equality
6 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) at the computer system, wherein f is a scalar
7 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
8 storing the representation in a memory within the computer system;

9 performing an interval global optimization process to compute guaranteed
10 bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the set of
11 equality constraints;
12 wherein performing the interval global optimization process involves,
13 applying term consistency to the set of equality constraints
14 over a subbox \mathbf{X} , and
15 excluding portions of the subbox \mathbf{X} that can be shown to
16 violate any of the equality constraints

1 9. The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 preconditioning the set of equality constraints through multiplication by an
4 approximate inverse matrix \mathbf{B} to produce a set of preconditioned equality
5 constraints;
6 applying term consistency to the set of preconditioned equality constraints
7 over the subbox \mathbf{X} ; and
8 excluding portions of the subbox \mathbf{X} that can be shown to violate any of the
9 preconditioned equality constraints.

1 10. The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$;
4 unconditionally removing from consideration any subbox for which
5 $\inf(f(\mathbf{x})) > f_bar$;
6 applying term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} ;
7 and
8 excluding portions of the subbox \mathbf{X} that violate the inequality.

1 11. The computer-readable storage medium of claim 8, wherein
2 applying term consistency involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term, $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$,
5 wherein the term $g(x_j)$ can be analytically inverted to produce an inverse function
6 $g^{-1}(y)$;
7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(X'_j) = h(\mathbf{X})$;
9 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
10 intersecting X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
11 wherein the new subbox \mathbf{X}^- contains all solutions of the equation within
12 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
13 the size of the subbox \mathbf{X} .

1 12. The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 applying box consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$
4 ($i=1, \dots, r$) over the subbox \mathbf{X} ; and
5 excluding portions of the subbox \mathbf{X} that violate the set of equality
6 constraints.

1 13. The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves:
3 evaluating a first termination condition;

4 wherein the first termination condition is TRUE if a function of the width
5 of the subbox **X** is less than a pre-specified value, ε_X , and the absolute value of the
6 function, f , over the subbox **X** is less than a pre-specified value, ε_F ; and
7 if the first termination condition is TRUE, terminating further splitting of
8 the subbox **X**.

1 14. The computer-readable storage medium of claim 8, wherein
2 performing the interval global optimization process involves performing an
3 interval Newton step on the John conditions.

1 15. An apparatus that solves a global optimization problem specified
2 by a function f and a set of equality constraints, the apparatus comprising:
3 a receiving mechanism that is configured to receive a representation of the
4 function f and the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$), wherein f is a
5 scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
6 a memory for storing the representation;
7 an optimizer that is configured to perform an interval global optimization
8 process to compute guaranteed bounds on a globally minimum value of the
9 function $f(\mathbf{x})$ subject to the set of equality constraints;
10 wherein the optimizer is configured to,
11 apply term consistency to the set of equality constraints
12 over a subbox **X**, and to
13 exclude portions of the subbox **X** that can be shown to
14 violate any of the equality constraints

1 16. The apparatus of claim 15, wherein the optimizer is configured to:

2 precondition the set of equality constraints through multiplication by an
 3 approximate inverse matrix **B** to produce a set of preconditioned equality
 4 constraints;
 5 apply term consistency to the set of preconditioned equality constraints
 6 over the subbox **X**; and to
 7 exclude portions of the subbox **X** that can be shown to violate any of the
 8 preconditioned equality constraints.

1 17. The apparatus of claim 15, wherein the optimizer is configured to:
 2 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$;
 3 unconditionally remove from consideration any subbox for which
 4 $\inf(f(\mathbf{x})) > f_bar$;
 5 apply term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox **X**;
 6 and to
 7 exclude portions of the subbox **X** that violate the inequality.

1 18. The apparatus of claim 15, wherein while applying term
 2 consistency, the optimizer is configured to:
 3 symbolically manipulate an equation to solve for a term, $g(x_j)$, thereby
 4 producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be
 5 analytically inverted to produce an inverse function $g^{-1}(y)$;
 6 substitute the subbox **X** into the modified equation to produce the equation
 7 $g(X'_j) = h(\mathbf{X})$;
 8 solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and to
 9 intersect X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;

10 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
11 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
12 the size of the subbox \mathbf{X} .

1 19. The apparatus of claim 15, wherein the optimizer is configured to:
2 apply box consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$)
3 over the subbox \mathbf{X} ; and to
4 exclude portions of the subbox \mathbf{X} that violate the set of equality
5 constraints.

1 20. The apparatus of claim 15, wherein the optimizer is configured to:
2 evaluate a first termination condition;
3 wherein the first termination condition is TRUE if a function of the width
4 of the subbox \mathbf{X} is less than a pre-specified value, ε_X , and the absolute value of the
5 function, f , over the subbox \mathbf{X} is less than a pre-specified value, ε_F ; and to
6 terminate further splitting of the subbox \mathbf{X} if the first termination
7 condition is TRUE

1 21. The apparatus of claim 15, wherein the optimizer is configured to
2 perform an interval Newton step on the John conditions.